Section A.

Qu 1: (i) (a) (ii) (a) (iii) (c)

(iv) (d) (v) (a) (vi) (d)

(vii) (a) (viii) (c) (ix) (d)

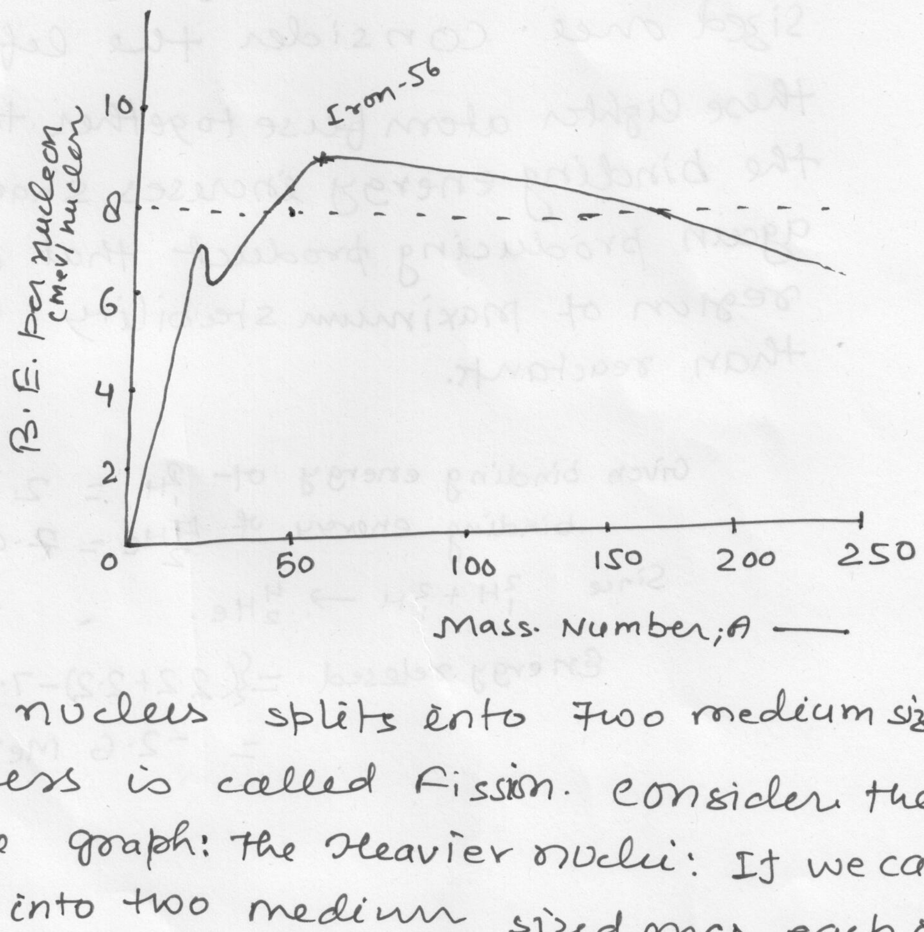
(x) (b)

Section B

Qu2: Marks (6+2)

Fission and Fusion:

To understand that under what conditions nuclear fission and nuclear fusion takes place, we need to consider the relative stability of heavy and light isotopes. In figure the binding energy per nucleon is plotted for the number of nucleons for various atomic nuclei. The higher the binding energy most stable the nucleus is. The graph has max value at 8.8 MeV/n when the no of nucleons is 56. This corresponds to 56 Fe, which is most stable nucleus since most energy is needed to pull a nucleon away from it.



When a heavy nucleus splits into two medium sized nucleus the process is called fission. Consider the right side of the graph: the heavier nuclei: If we can some how split into two medium sized ones, each of the new nuclei will have more binding energy per nucleon then the original once.

The nuclei of the daughter atoms typicaly have mass number that av. around 118, which is closer to region of max stability. For example It the Uranium $^{235}_{92}U$ is broken into smaller nuclei the B.E. differency per nucleon is about 0.8 MeV. The total energy given off is therefore

$$\left(0.8\ \frac{\text{MeV}}{\text{nucleon}}\right)(235\ \text{nucleons}) = 188\ \text{MeV}.$$

This enormous amount of energy to be produced in a single atomic event. However ordinary chemical reaction librate only a few eV per spacing atom.

The binding energy of the daughter nuclei of a fisson reaction is therefore grater than that of parent. Thus products are more stable than reactants.

Fussion is the process in which, ~~when we~~ joing of two light nuclei together gives a single nuclees of medium sized once. Consider the left side of graph, when these lighter atom fuse together to form a heaviear nucleus, the binding energy incresses sharply. ~~whi~~ With the reaction again producing product that is closer to to the region of maximum stability. The product is more stable than reactants.

Given binding energy of ^2_1H = 2.2 MeV

binding energy of ^4_2He = 7.0 MeV

Since $^2_1H + ^2_1H \rightarrow ^4_2He$.

Energy relesed $= \{(2.2+2.2)-7.0\}$ MeV

$= -2.6$ MeV

Qu 3: Total magnetic moment is sum of orbital and intrisic moment

$$\bar{\mu} = \bar{\mu}_L + \bar{\mu}_S \quad — (1)$$

where, $\mu_L \rightarrow$ orbital mag. moment

$\mu_S \rightarrow$ spin mag. moment, which is vector sum of intrisic mag. moment of individual nucleons in the nucleus.

For proton and neutron the intrinsic moments are

$\mu_p = 2.79$ nm and $\mu_n = -1.91$ nm

In case of odd Z nuclei, mag. moment is given by last unpaired proton.

Therefore, orbital contribution due to proton orbital ang. momentum $(\bar{l})$ is

$$= \frac{e\hbar}{2m}\bar{l}$$

Intrisic mag. moment $= (2\bar{S})\vec{\mu}_p$

$\therefore$ Total mag. moment is

$$\bar{\mu} = (\bar{l} + \bar{\mu}_p(2\bar{S}))\ \text{nm}.$$

Since $\bar{l}$ and $\bar{S}$ process around $\bar{J}$, so also $\bar{\mu}$. Also J is precessing about the quantization (magnetic field)

The observed mag. moment is given by—

$$\bar{\mu}_{obs} = \frac{\langle \bar{\mu}\cdot\bar{J}\rangle}{\sqrt{J(J+1)}} \cdot \frac{J}{\sqrt{J(J+1)}}$$

$$\bar{\mu}_{obs} = \frac{\langle \bar{\mu}\cdot\bar{J}\rangle}{J+1} \quad — (1)$$

$$\therefore \bar{\mu}_{obs} = \frac{1}{J+1}\left[\langle \bar{l}\cdot\bar{J}\rangle + 2\mu_p\langle \bar{S}\cdot\bar{J}\rangle\right]\ \text{nm}$$

Since $\bar{J} = \bar{l} + \bar{S}$, $\bar{S} = \bar{J} - \bar{l}$ so, $S^2 = \bar{J}^2 + \bar{l}^2 - 2\bar{J}\cdot\bar{l}$

$$\therefore \langle \bar{J}\cdot\bar{l}\rangle = \frac{J(J+1) + l(l+1) - S(S+1)}{2}$$

Taking $S = 1/2$

$$\langle \bar{l}\cdot\bar{J}\rangle = \frac{J(J+1) + l(l+1) - 3/4}{2}$$

Again $J = \bar{l} + \bar{S}$, $l = \bar{J} - \bar{S}$

$$\therefore \langle \bar{S}\cdot\bar{J}\rangle = \frac{J(J+1) \cancel{+} l(l+1) + 3/4}{2}$$

Therefore,

$$\bar{\mu}_{obs} = \frac{1}{J+1}\left[\frac{J(J+1)+l(l+1)-3/4}{2} + \mu_p\left\{J(J+1)-l(l+1)+3/4\right\}\right] \quad -(2)$$

Now for, $\bar{J} = l + 1/2$, $l = J - 1/2$

$$\therefore \bar{\mu}_{obs} = \frac{1}{J+1}\left[\frac{J(J+1)+(J-1/2)(J-1/2+1)-\frac{3}{4}}{2} + \mu_p\left\{J(J+1)-(J-1/2)(J-1/2+1)+\frac{3}{4}\right\}\right]$$

$$\bar{\mu}_{obs} = \frac{1}{J+1}\left[\frac{(J+1)(2J-1)}{2} + \mu_p(J+1)\right]$$

$$= \frac{(2J-1)}{2} + \mu_p = \left(J - \tfrac{1}{2} + 2.79\right) nm.$$

$$\bar{\mu}_{obs} = (J + 2.29)\ nm \quad - (3)$$

For $L = J + 1/2$

$$\bar{\mu}_{obs} = \frac{1}{J+1}\left[\frac{J(J+1)+(J+1/2)(J+1/2+1)\frac{3}{4}}{2} + \mu_p\left\{J(J+1)-(J+\tfrac{1}{2})(J+\tfrac{1}{2}+1)+\frac{3}{4}\right\}\right]$$

$$= \frac{1}{J+1}\left[J(J+3/2) - \mu_p . J\right]$$

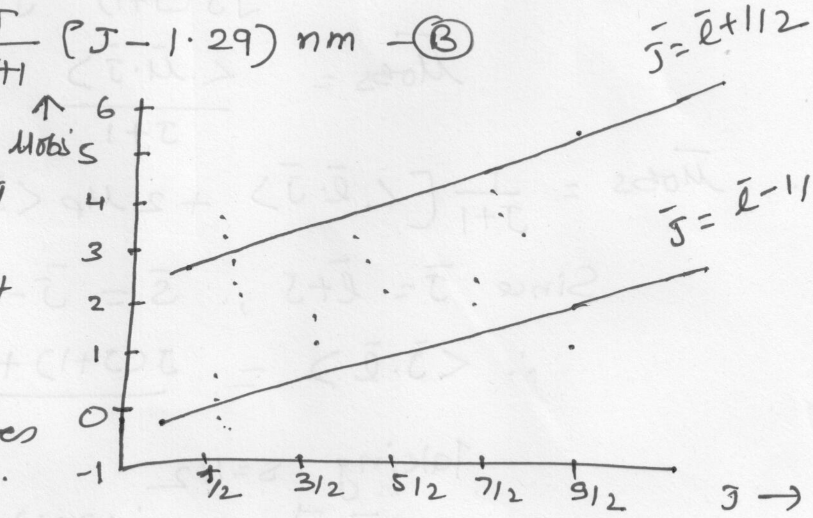
$$= \frac{J}{J+1}\left[J + \frac{3}{2} - 2.79\right] nm \quad - (4)$$

<u>Hence</u>

$$\bar{\mu}_{obs}\ (J = l + 1/2) = (J + 2.29)\ nm \quad (A)$$

$$\bar{\mu}_{obs}\ (J = l - 1/2) = \frac{J}{J+1}(J - 1.29)\ nm \quad - (B)$$

These relation (A) & (B) defines two curves for J vs μ corresponding to $J = l+1/2$ and $J = l - 1/2$ values. The value of μ is known as Schmidt value and the curve are known as Schmidt lines. The value of mag. moment lies in between these two lines.



<u>Mag. Moment of ^{17}N and ^{19}O</u>

$^{17}_{7}N_{10}$ (odd Z) $= 1s^2_{1/2}\, 1p^4_{3/2}\, 1p^1_{1/2}$, $J = 1/2, l = 1$, $\mu = \cancel{\frac{1}{2} + 2.29 = 2.79\ nm}$

$\mu = \frac{J}{J+1}(J - 1.29)\ nm = \frac{\frac{1}{2}}{(\frac{1}{2}+1)}(\frac{1}{2} - 1.29) = -0.26\ nm$ for $J = l - 1/2$

$^{19}_{8}O_9$ = (odd N) $= 1s^2_{1/2}\, 1p^4_{3/2}\, 1p^2_{1/2}\, 1d^1_{5/2}$, $J = 5/2$ $l = 2$, So $\mu_n = -1.91\ nm$ for $J = l + 1/2$.

Qu4: In order to explain the saturation of nuclear forces, Heisenberg proposed that nuclear forces were exchange forces which would depend explicitly on the symmetry of the wave function. It was observed that π-mesons was being exchanged between nucleons. There are four kinds of exchange forces like—

(i) Wigner forces
(ii) Majorana forces
(iii) Bartlett or spin exchange force
(iv) Heisenberg forces.

The wave equation in the C-system can be written as -

$$\frac{\hbar^2}{2\mu}\nabla^2\psi + E\psi = V\psi \quad -(1)$$

Here, $\mu \to$ reduced mass and

$\psi = \psi(\bar{r}_1, \bar{r}_2)\, \chi(\sigma_1, \sigma_2) = \psi_{12}\chi_{12}$ is a function of the position and spin-co-ordinates of the two particles..

the effect of wavefunction in different force can be understand as follows—

(I) Wigner Forces:

Here, $V = V(r)\, \hat{P}_W$,

$\hat{P}_{(W)} = 1.$ for all states. and is known as Wigner operator.

$\therefore$ Wave equation becomes.

$$\left[\frac{\hbar^2}{2\mu}\nabla^2 + E\right]\psi_{12}\chi_{12} = V(r)\,\hat{P}_W\,\psi_{12}\,\chi_{12}.$$
$$= V(r)\,\psi_{12}\,\chi_{12} \quad -(2)$$

$\therefore$ The force is known as no exchange or Wigner force and the force is always attractive in nature.

(II) Majorana Force:

Here, $V = V(\bar{r})\,\hat{P}_m$,

P_m is Majorana operator

$$\hat{P}_m\,\psi_{12}\,\chi_{12} = \hat{P}_m\,\psi(\bar{r}_1\bar{r}_2)\,\chi(\sigma_1,\sigma_2)$$
$$= \psi(\bar{r}_2,\bar{r}_1)\,\chi(\sigma_1\,\sigma_2)$$
$$= \psi_{21}\,\chi_{12}$$

For two body problem, the exchange of two particles implies about the origin

$$\therefore \hat{P}_m \psi_{12} \chi_{12} = (-1)^L \psi_{12} \chi_{12}, \quad L \to \text{Ang. momentum.}$$

$\therefore$ The Schrodinger eqn becomes.

$$\left(\frac{\hbar^2}{2\mu} \nabla^2 + E\right) \psi_{12} \chi_{12} = V(r) \hat{P}_m \psi_{12} \chi_{12}$$

$$= (-1)^L V(r) \psi_{12} \chi(1,2) \quad -(3)$$

$\therefore$ For even parity Majorana Force are attractive and for odd parity it is negative.

(iii) Bartlett Force:

Here, $V(\bar{r}) = \hat{P}_B V(r)$, where $\hat{P}_B$ is the spin exchange operator.

$$\therefore \hat{P}_B \psi_{12} \chi_{12} = \hat{P}_B \psi(\bar{r}_1 \bar{r}_2) \cdot \chi(\sigma_1 \sigma_2).$$

$$= \psi(\bar{r}_1 \bar{r}_2) \chi(\sigma_2 \sigma_1)$$

$$= (-1)^{S+1} \psi_{12} \chi_{12}$$

The wave function becomes,

$$\left[\frac{\hbar^2}{2\mu} \nabla^2 + E\right] \psi_{12} \chi_{12} = V(r) \hat{P}_B \psi_{12} \chi_{12}.$$

$$= (-1)^{S+1} \psi_{12} \chi_{12} \quad -(4)$$

For the triplet state $(S=1)$, the spin function χ_{12} is symmetric and for the singlet state $(S=0)$ the spin wave function is antisymmetric

(iv) Heisenberg Forces:

Here, $V = V(r) \hat{P}_H$, where $\hat{P}_H$ is Heisenberg Operator

$$\hat{P}_H \psi_{12} \chi_{12} = (-1)^{L+S+1} \psi_{12} \chi_{12}$$

$\therefore$ the wave eqn becomes.

$$\left[\frac{\hbar^2}{2\mu} \nabla^2 + E\right] \psi_{12} \chi_{12} = V(r) \hat{P}_H \psi_{12} \chi_{12}$$

$$= (-1)^{L+S+1} V(r) \psi_{12} \chi_{12} \quad -(5)$$

$$= +V(r) \psi_{12} \chi_{12}$$

for $L = 0$, even, $S = 1$. or $L = \text{odd}$, $S = 0$

$$= -V(r) \psi_{12} \chi_{12}$$

for. $L = 0$, even ; $S = 0$, $L = \text{odd}$; $S = 1$

Thus, the Heisenberg force are attractive for even triplet states and odd singlet states. However, repulsive for odd triplet or even state

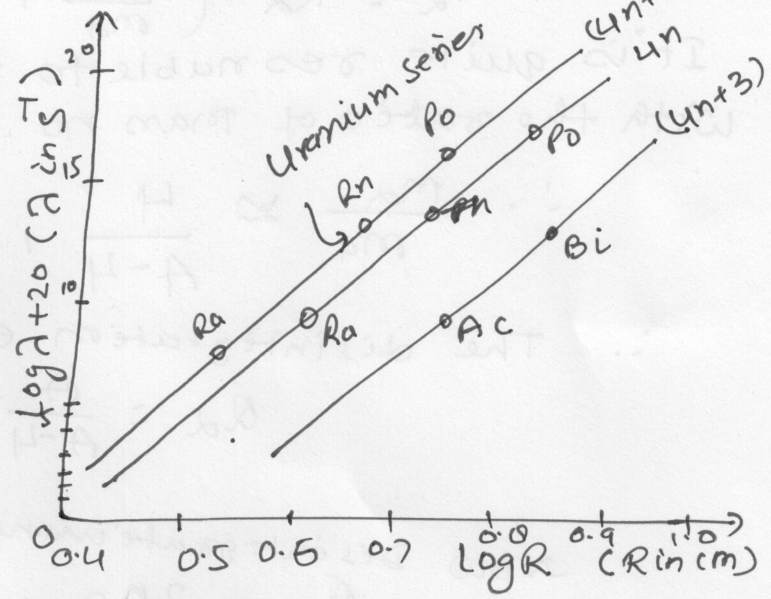
Qu5: (3+3+2)

<u>Geiger - Nuttal Law</u>: There is an emperical relation between the range (R) of an alpha particle and the decay constant λ $(\frac{1}{\tau})$ of the α-emitter

$$\log R = A \log \lambda + B \quad — (1)$$

Where, A and B are constants. The constant A has the same value for all three radioactive series and B is a constant with different value for each series.

The relation (1) is known as Geiger Nuttall Law and is plotted for three radioactive series. The Geiger-Nuttal law is very important because it estimate half lives of α-emitters which could not be easily determind by experimental measurements.



<u>Relation between disintegration energy and α-particle energy</u>

Typical α-decay can be written as.

$$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He + Q_\alpha \quad — (1)$$

Where, Q_α is total energy released in decay process and known as disintegration energy.

Suppose mass of parent is m_p, that of the daughter is m_d and that of α-particle is m_α. When an α-particle is emitted with velocity V_α the residual daughter nuclei will recoil with velocity V_d such that.

$$m_d V_d = m_\alpha V_\alpha \quad \text{by conservation of momentum}$$

4. $$Q_\alpha = (\text{Final K.E}) - (\text{Initial K.E})$$

$$= \left(\frac{1}{2} m_d V_d^2 + \frac{1}{2} m_\alpha V_\alpha^2\right) - 0$$

$$= \frac{1}{2} m_d \left(\frac{m_\alpha V_\alpha}{m_d}\right)^2 + \frac{1}{2} m_\alpha V_\alpha^2$$

$$= \frac{1}{2} m_\alpha V_\alpha^2 \left(\frac{m_\alpha}{m_d} + 1\right)$$

Putting $\frac{1}{2} m_\alpha V_\alpha^2 = K_\alpha$, we have

$$Q_\alpha = K_\alpha \left(\frac{m_\alpha}{m_d} + 1\right)$$

It is quite resnuble to replace the ratio of masses with the ratio of mass no.

$\therefore \frac{m_\alpha}{m_d} \simeq \frac{4}{A-4}$, A is mass of parrent nuclei

$\therefore$ The disintegration energy becomes.

$$Q_\alpha = \frac{A}{A-4} K_\alpha \quad \text{———} \quad (A)$$

Now Disintegrate energies —

$$Q_{\alpha_1} = \frac{240}{236} \times 5.17 = 5.25 \text{ MeV}$$

$$Q_{\alpha_2} = \frac{240}{236} \times 5.02 = 5.10 \text{ MeV}$$

Energy of emitted gamma ray $= 0.15$ MeV (4+4)

Qus! N. Bohr proposed the machanisum of ~~of~~ compound nucleus (C.N) Compound nucleus formation is basically two step process

I[st] Step! The incedent particle is absorbed by the target nucleus forming an intermediate stage called C.N. which remains for long time ($\sim 10^{-16}$ s) compare to the small time ($\sim 10^{-22}$ sec) required for direct reaction. During this time the kinectic energy of incedent particle is shared among all the nucleons. All memory of incedent particle and target is lost. The C.N. so formed is always in highly excited unstatable state.

At this stage statistical equilibrium is reached. Once equilibrium is reached the C.N. retaines no memory of its mode of formation.

2nd step: The emisson takes place through statistical fluctuation from equilibrium through exit channels. This is achived by emisson of charged particle and gamma rays.

Such reaction have definite intermediate state, after absorption of encedent particle. This intermedeate stage is called compound nucleus and represented as –

$$a + X \rightarrow C^* \rightarrow Y + b.$$

Where C^* is C.N.

The decay of C^* depends only on properties of C^* and not up on how it was formed. Actully C^* has long life time, so by the time it is ready to breakup it forgets as to how it was formed.

The decay probability of C.N. is equal to reciprocal of mean lifetime of compound nucleus. If Γ is width of the level then.

$$\Gamma = \frac{\hbar}{\tau}$$

Let us consider specific example of decay of C.N. The C.N. $^{64}Zn^*$ can be formed through several reations including $p + {}^{63}Cu$ & $\alpha + {}^{60}Ni$. It can decays in varity of ways including $^{63}Zn + n$, $^{62}Zn + 2n$ and $^{62}Cu + p + n$ –

$$\begin{matrix} p + {}^{63}Cu \\ \alpha + {}^{60}Ni \end{matrix} \rightarrow {}^{64}Zn \rightarrow \begin{matrix} {}^{63}Zn + n \\ {}^{62}Cu + n + p \\ {}^{62}Zn + 2n. \end{matrix}$$

<u>Kineticlnegy of α- particle:</u>

$$^{14}N\,(\alpha, p)\,^{17}O$$

$$^{14}_{7}N + {}^{4}_{2}He \rightarrow {}^{17}_{8}O + {}^{1}_{1}H + Q.$$

$$14.003074 \quad 4.002604 \quad 16.99913 \quad 1.007833$$

$$Q = M_X + M_a - M_b - M_y$$

$$= [14.003074 + 4.002604 - 1.00783 - 16.99913]\,u$$

$$= (18.006578 - 18.00696)\,u \times 931.5\ \frac{MeV}{u}$$

$$Q = -1.20\ MeV$$

$\Rightarrow$ Reaction will endo-ergic reaction.

<u>Now</u>, $E_{th} = -Q\left[1 + \frac{M_a}{M_X}\right]$

$$= -(1.20)\left[1 + \frac{4.002604}{14.003074}\right] 931.5\ MeV.$$

$$E_{th} = 1.54\ MeV \quad \Rightarrow \text{Min. K.E to cause the reaction}$$

<u>Qu:7</u>: <u>Gell-Mann-Okubo Formula</u>: (6+2)

Since mass of the different member of an SU(3) multiplet is not same, Hence Okubo put fourth the following formula for the mass of member withen multiplet -

$$M(Y,T) = M_0 + \alpha Y + b\left[T(T+1) - \frac{1}{4}Y^2\right] \quad \text{— (1)}$$

Where, M_0, α, b, are constant withen any particular multiplet. The above formula is called Gellmann-Okubo formula. This is applicable to baryons. But in case of mesons for better agrement with experimental value the M is replace by M^2 in eqn (1).

For baryon octet $J^\pi = \frac{1}{2}^+$

the experimental value are in good agrement with the following formula.

$$2(M_N + M_\Xi) = (3 \cdot M_\Lambda + M_\Sigma) \quad \text{— (2)}$$

Where, M_N is average nucleon mass, M_Σ is av. mass of Σ hyprons and M_Λ is the mass of Λ. hypron.

The Gell-Mann-Okubo mass formula is excelent agrement with the baryan decoupled with $J^{\pi} = \frac{3}{2}^{+}$.

For this decouplet we have.

$$T = 1 + \frac{1}{2}Y \quad (3)$$

Using eqn (3) eqn (1) becomes.

$$M(Y,T) = M_0 + \alpha Y + b\left[(1+\tfrac{1}{2}Y)(2+\tfrac{1}{2}Y) - \tfrac{1}{4}Y^2\right]$$

or

$$M(Y,T) = (M_0 + 2b) + (\alpha + \tfrac{3}{2}b)Y. \quad -(4)$$

This formula predicts equal mass spacing in the decouplet which is in agremet with observed mass. Thus we have.

$$(M_\Sigma - M_\Lambda) = (M_\Xi - M_\Sigma) = (M_\Omega - M_\Xi) \quad -(5)$$

For the mesons multiplet on SU(3) classificati on scheme Gell-Mann, obtained the following mass formulla by applying eqn (1) to the sq. of meson masses.

$$M_K^2 = \frac{1}{4}(3M_\eta^2 + M_\pi^2)$$

this agrees well with experimental value.

Mass of Ω^- particle :

From eqn (5),

$$(M_\Omega - M_\Xi) = (M_\Xi - M_\Sigma) = (M_\Sigma - M_\Delta) = -\alpha - \frac{3}{2}b$$

Since

$$M_\Sigma = 1385 \text{ MeV}$$

$$M_\Xi = 1530 \text{ MeV}$$

$$M_\Omega \simeq 2M_\Xi - M_\Sigma \simeq 1675 \text{ MeV}$$

The actual mass of omega meson was found to be 1672.5 MeV. This agrees well with mass formula.

Que 8: ① Reciprocity theoram:

Consider a reaction

$$a + X \rightarrow Y + b.$$

Let us denote the entrance chanel $(a+X)$ by α and exit β.

$\therefore$ The process can be written as

$$\alpha \rightarrow \beta$$

and inverse of above process will be

$$\beta \rightarrow \alpha$$

Suppose particle a, x, Y, b are kept in box of volum V and both the process takes place ie

$$\alpha \rightleftharpoons \beta$$

There is a theoram in statistics known as "Principle of overall ballence" say that

$$\frac{\text{No of existing } \alpha \text{ states in } dE \text{ say } N_\alpha}{\text{No of existing } \beta \text{ states in } dE \text{ say } N_\beta} = \frac{\text{No of possible } \alpha \text{ states in } dE}{\text{No of possible } \beta \text{ states in } dE}$$

$\therefore$ No. of free particles state in momentum range p and $p+dp$ in box of volume V are -

$$N = \frac{4\pi p^2 V dp}{h^3} = \frac{p^2 V dp}{2\pi^2 \hbar^3} \qquad \text{as. } \hbar = \frac{h}{2\pi}, \quad h = 2\pi\hbar$$

$$\therefore N_\alpha = \frac{p_\alpha^2 V dp_\alpha}{2\pi^2 \hbar^3}$$

Also $v\, dp = dE$ $\rightarrow$ $v \rightarrow$ velocity.

$$\therefore N_\alpha = \frac{p_\alpha^2 dE_\alpha}{2\pi^2 \hbar^3 . v_\alpha} \qquad \text{Similarly } N_\beta = \frac{p_\beta^2 dE_\beta}{2\pi^2 \hbar^3 v_\beta}$$

The energy range for the two channels must be same ie

$$dE_\alpha = dE_\beta$$

$$\therefore \boxed{\frac{N_\alpha}{N_\beta} = \frac{p_\alpha^2 v_\beta}{p_\beta^2 v_\alpha}} \quad \text{— } (1)$$

The system is in dynamical equilibrium when No of $\alpha \rightarrow \beta$ transition per second is equal to No of $\beta \rightarrow \alpha$. transitions per sec. this condition is called Principle of detailed Balence

$\therefore$ No of transitions ($\alpha \to \beta$) per sec = No of transition ($\beta \to \alpha$) per sec

$$\Rightarrow \quad N_\alpha T(\alpha \to \beta) = N_\beta T(\beta \to \alpha)$$

where $T(\alpha \to \beta)$ is transition probability for transition ($\alpha \to \beta$).

$$\therefore \quad \frac{N_\alpha}{N_\beta} = \frac{T(\beta \to \alpha)}{T(\alpha \to \beta)} \quad \text{—— (2)}$$

Let $\sigma \to$ Scattering cross-section.

$$\therefore \quad T(\alpha \to \beta) = \frac{\sigma(\alpha \to \beta) v_\alpha}{V} \quad \& \quad T(\beta \to \alpha) = \frac{\sigma(\beta \to \alpha) v_\beta}{V}$$

$$\therefore \quad \frac{N_\alpha}{N_\beta} = \frac{\sigma(\beta \to \alpha) v_\beta}{\sigma(\alpha \to \beta) v_\alpha} \quad \text{—— (3)}$$

From (1) & (3).

$$\frac{p_\alpha^2 v_\beta}{p_\beta^2 v_\alpha} = \frac{\sigma(\beta \to \alpha) v_\beta}{\sigma(\alpha \to \beta) v_\alpha}$$

$$\Rightarrow \quad p_\alpha^2 \, \sigma(\alpha \to \beta) = p_\beta^2 \, \sigma(\beta \to \alpha).$$

This is called reciprocity theorem.

(ii) <u>Spin dependence of Nuclear Forces</u>

Let us consider the ground state with accurately known binding energy, ie $E_1 = 0$ and $E_2 = -E_d$ [Deutron case]

The ground state of deutron,

$$V_2 = e^{-\alpha r} \quad \text{as} \quad V_2(r=0) = 1.$$

$$\text{where} \quad \alpha = \frac{M E_d}{\hbar^2}$$

$$V_2' = -\alpha . e^{-\alpha r}$$

From Effective range theory —

$$-(V_2 V_1' - V_2' V_1)\big|_{r=0} = (k_2^2 - k_1^2)\int_0^\infty (V_1 V_2 - U_1 U_2)\, dr$$

Put $k_1 = 0$, & $k_2 = k$.

$$-\frac{1}{a(0)} - (-\alpha) = -\alpha^2 \int_0^\infty (U U_0 - V V_0)\, dr$$

$$-\frac{1}{a(0)} + \alpha = \alpha^2 \int_0^\infty (V_0^2 - U_0^2)\, dr$$

$$\frac{1}{a(0)} = \alpha - \frac{\alpha^2 r_0}{2} \; ; \; r_0 = \frac{1}{2}\int_0^\infty (v_0^2 - u_0^2)\, dr \quad -(1)$$

Now scattering cross-section,

$$\sigma = \frac{4\pi}{k^2 + \frac{1}{a^2(k)}}$$

$$\therefore \sigma = \frac{4\pi}{k^2 + \left(\frac{1}{a(0)} - \frac{1}{2} r_0 k^2\right)^2} = \frac{4\pi}{k^2 + \left[\alpha - \frac{1}{2}\alpha^2 r_0 - \frac{1}{2}k^2 r_0\right]^2}$$

$$\Rightarrow \sigma = \frac{4\pi}{k^2 + \left[\alpha - \frac{r_0}{2}(k^2+\alpha^2)\right]^2} = \frac{4\pi}{k^2 + \left[\alpha^2 - r_0(k^2+\alpha^2)\alpha + \frac{1}{4}r_0^2(k^2+\alpha^2)^2\right]}$$

The zero energy cross-section (ie $k=0$) given by.

$$\sigma_0 = \frac{4\pi}{\alpha^2\left(1 - r_0\alpha + \frac{1}{4}r_0^2\alpha^2\right)}$$

$$= \frac{4\pi}{\alpha^2\left[1 - \frac{1}{2}r_0\alpha\right]^2}$$

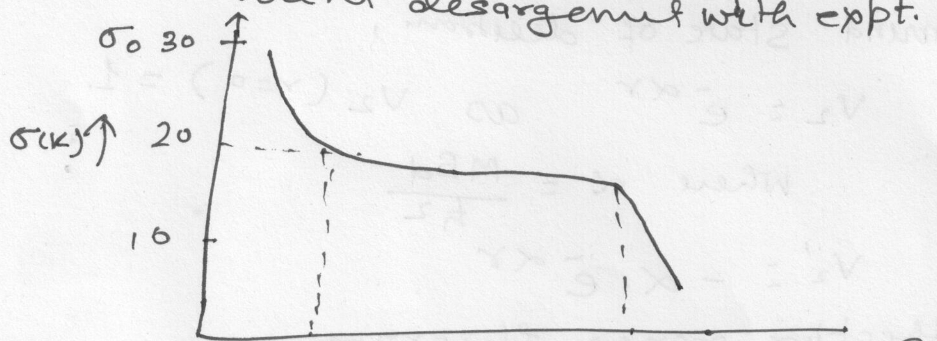
For, σ_0 to be max, $r_0 = \frac{1}{\alpha}$ ie $\sigma_0\big|_{max} = 4\times\frac{4\pi}{\alpha^2} = 9.32$ barns

$\sigma_0|_{min} = \frac{4\pi}{\alpha^2} = 2.32$ b.

Thus $2.32\text{ b} \le \sigma_0 \le 9.32$ b.

However, $\sigma_0|_{exp} = 20.36 \pm 0.10$ b

Which is inviolent desargement with expt. values.



This disagreement is sign of sum fundamental error in our assumptions. This point is cleared by Wigner. He suggested that scattering occurs not only in the ~~singlet stat~~ triplet state but in singlet state as well. Hence total scattering is given by –

$\sigma = \frac{3}{4}\sigma_T + \frac{1}{4}\sigma_S$; where σ_S is scattering cross-section for single state and σ_t for triplet state, $\frac{1}{4}$ & $\frac{3}{4}$ are st. wt factor

Now $\sigma_T = 2.32$b and $\sigma_0 = 20.36$b then

$\sigma_S = 74.6$ b.